

Interest Rate Hedge

Extension of a Rho Hedge

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1 Rationale

Currently risks originating from moves in interest rates are often hedged using a simple Rho hedge. This provides a reasonable protection against parallel moves of the yield curve but none at all against twists and partial shifts.

Examples: If one would have invested 1,000,000€ at the beginning of 2005 in an asian call option with a maturity of six years and would not perform any hedges against interest rate moves, the value of the unhedged position would have changed by 22,000€¹ based only on the moves in the Euro yield curve. If one would have used a simple Rho hedge the change in value would even have been 40,100€. The picture repeats for a Down-and-out put with a maturity of 35 months: without any interest rate hedge 11,600€, with Rho hedge 25,000€.

The reason for this extraordinary poor hedging result seems to be the kind of interest rate move: in 2005 the Euro yield curve twisted around the 5-years interest rate, i.e. interest rates for shorter maturities rose and for longer maturities fell.

The existing approach using a Rho hedge should be extended by risk figures which provide additional protection against moves in the yield curve. Following the term *tau* will be used for such risk figures.

In the following section multiple approaches for the additional risk figures are presented and tested against hypothetical scenarios as well as historical yield curve movements.

All the following considerations refer to the Euro yield curve. For sake of simplicity it is assumed that Euro-Schatz, Euro-BOBL and Euro-Bund can be traded directly and are used as hedging instruments (furthermore referred to as „hedging bonds“). Adoption to different currencies and different hedging instruments would be trivial by adjusting the time horizon of the presented risk figures. The only condition that must be fulfilled is that the hedging instruments and the derivatives which should be hedged are based on the same yield curve.

¹ In this paper the sign of any value changes is ignored as only the absolute change (negative or positive) matters when hedging against any price risk.

2 Approaches for additional risk figures

In this section different approaches for the determination of additional risk figures are presented. In general all risk figures are calculated by valuating the respective derivative with the reference yield curve (see figure 1), a second valuation with a modified yield curve and finally by calculating the difference quotient. A total of three risk figures will be determined. If they fulfill the properties shown in table 1, a simple linear equation system can be used to calculate the needed hedge positions to set all determined risk figures to zero and therefore establish a hedge against interest rate moves.

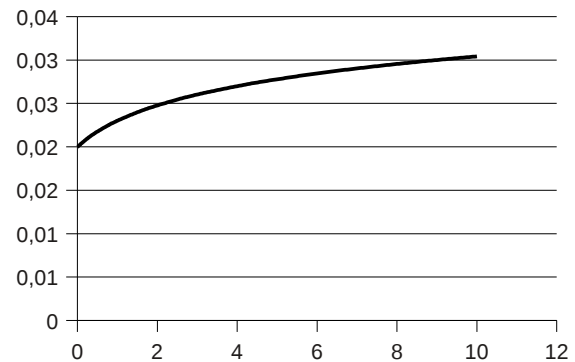


Figure 1: Reference yield curve

	Tau_1	Tau_2	Tau_3
Euro-Schatz	Different from zero	Zero	Zero
Euro-BOBL	Different from zero	Different from zero	Zero
Euro-Bund	Different from zero	Different from zero	Different from zero

Table 1: Preferable properties of the risk figures

To verify the different approaches the risk figures will be calculated based on the reference yield curve for some derivatives as well as the hedging bonds and a portfolio of the respective derivative and the hedging bonds will be created in such a way that all three risk figures will amount to zero. Subsequent the yield curve is modified and the portfolio is revalued. The change in the value of the portfolio is then used as measure for the quality of the interest rate hedge.

The following derivatives are used for verification:

- A) Asian call option with a remaining maturity of six years and annual averaging
- B) Winner Certificate (i.e. a structure of a coupon bond and an asian call option) with a remaining maturity of eight years. The first averaging will is scheduled in eight months; all the remaining averaging dates are scheduled eleven months apart each. At the averaging dates also a fixed coupon will be paid..
- C) Asian call option with a remaining maturity of eighteen months and monthly averaging.
- D) European call option with a remaining maturity of twenty five months.
- E) Coupon bond with a remaining maturity of seven years and annual coupon.
- F) Zero bond with a remaining maturity of three years.
- G) Down-and-out put option with continuous monitoring and a remaining maturity of thirty five months; valuated using the Derman-Ergener-Kani replication approach.

The following table lists all yield curve modifications which are used during the verification:

No	Name	Description
Z1	Rates +1%	The reference yield curve is parallel shifted upwards by 1%.
Z2	Rates +5%	The reference yield curve is parallel shifted upwards by 5%.
Z3	Lifting short	The reference yield curve is modified in such a way, so that the overnight rate is increased by 1%, the ten-year rate remains unchanged and rate changes in between are linear.
Z4	Lifting long	The reference yield curve is modified in such a way, so that the ten-year rate is increased by 1%, the overnight rate remains unchanged and rate changes in between are linear.
Z5	Twist 2 years	The reference yield curve is modified in such a way, so that the two-year rate remains unchanged, the ten-year rate is reduced by 0.4% and the overnight rate is increased by 0.25%; rate changes in between are linear.
Z6	Flat 3%	For all maturities the interest rate is 3%.
Z7	Flat 2%	For all maturities the interest rate is 2%.
Z8	Flat 5%	For all maturities the interest rate is 5%.
Z9	Steep increase	The reference yield curve is modified in such a way, so that the ten-year rate is increased by 5%, the overnight rate remains unchanged and rate changes in between are linear.
Z10	Inverse	The reference yield curve is modified in such a way, so that the five-year rate remains unchanged, the ten-year rate is reduced by 0.5% and the overnight rate is increased by 1%; rate changes in between are linear.

For comparison the value changes of the derivatives are calculated if they remain unhedged resp. if they are hedged using a simple rho hedge with Euro-Bund. Following the results for the unhedged case:

	A	B	C	D	E	F	G
Z1	+8.34%	-4.72%	+5.43%	+7.45%	-6.06%	-2.96%	-6.02%
Z2	+41.22%	-19.68%	+28.34%	+39.48%	-26.64%	-13.93%	-31.22%
Z3	+5.58%	-0.45%	+4.93%	+5.88%	-2.05%	-2.08%	-3.71%
Z4	+2.77%	-4.33%	+0.49%	+1.53%	-4.10%	-0.90%	-2.27%
Z5	-0.52%	+1.77%	+0.73%	-0.03%	+1.48%	+0.15%	0.64%
Z6	+3.13%	+0.12%	+3.93%	+3.77%	-0.75%	-1.19%	-1.70%
Z7	-5.20%	+5.29%	-1.47%	-3.55%	+5.69%	+1.82%	4.20%
Z8	+19.91%	-8.82%	+15.10%	+19.15%	-12.40%	-6.94%	-14.20%
Z9	+13.63%	-19.04%	+2.45%	+7.76%	-18.68%	-4.40%	-11.34%
Z10	+2.63%	+2.15%	+4.44%	+4.31%	+1.00%	-1.19%	-1.41%

The following tables shows the results in case of a rho hedge, whereas all situations which show greater changes in value than the unhedged case are marked in red.

	A	B	C	D	E	F	G
Z1	+0.36%	+0.02%	+0.27%	+0.41%	-0.07%	-0.08%	-0.35%
Z2	+7.68%	+0.22%	+6.65%	+9.87%	-1.42%	-1.84%	-7.41%
Z3	+4.87%	-0.03%	+4.48%	+5.25%	-1.52%	-1.82%	-3.21%
Z4	-4.52%	-0.01%	-4.22%	-4.90%	+1.38%	+1.73%	2.90%
Z5	+2.51%	-0.03%	+2.69%	+2.65%	-0.80%	-0.94%	-1.51%
Z6	+3.08%	+0.15%	+3.89%	+3.73%	-0.71%	-1.17%	-1.67%
Z7	+3.51%	+0.12%	+4.16%	+4.13%	-0.86%	-1.32%	-1.98%
Z8	+4.58%	+0.27%	+5.18%	+5.62%	-0.87%	-1.41%	-3.31%
Z9	-16.94%	-0.91%	-17.32%	-19.23%	+4.30%	+6.63%	10.36%
Z10	+6.08%	+0.10%	+6.67%	+7.36%	-1.60%	-2.44%	-3.86%

2.1 Parallel

Risk figure	Modification of the yield curve
Tau₁	Parallel shift of 0.01% upwards from zero (incl.) to two (incl.) years.
Tau₂	Parallel shift of 0.01% upwards from two (excl.) to five (incl.) years.
Tau₃	Parallel shift of 0.01% upwards from five (excl.) to ten (incl.) years.

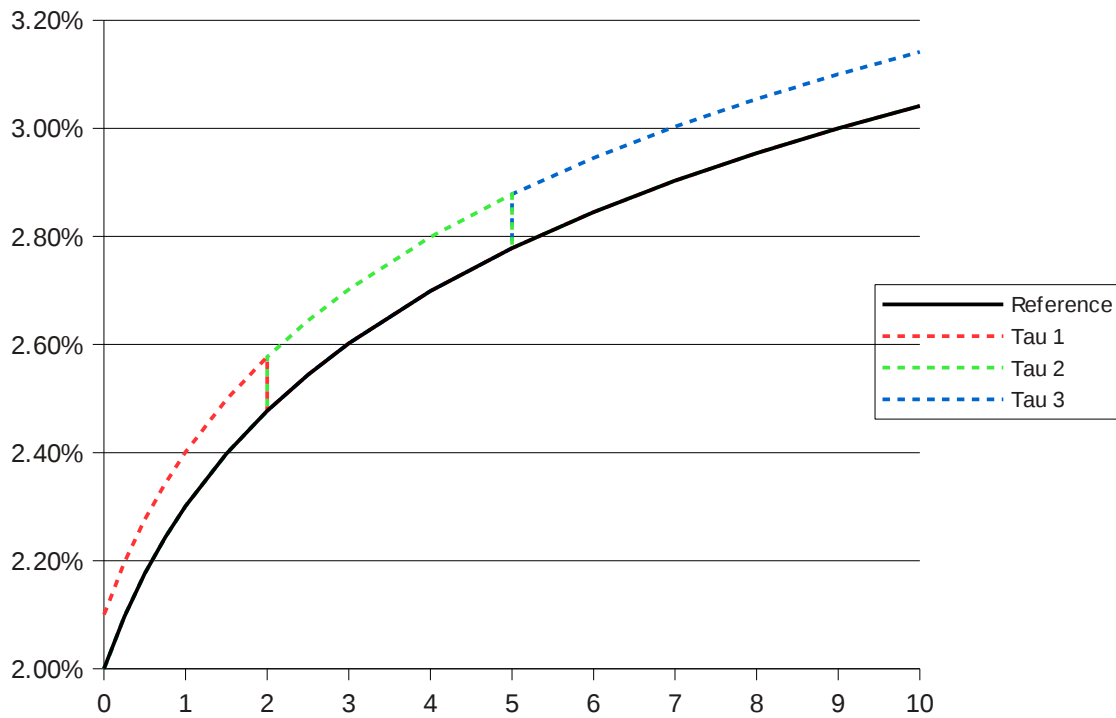


Figure 2: Partial parallel shift

	A	B	C	D	E	F	G
Z1	+0.17%	-0.01%	+0.12%	+0.30%	-0.07%	-0.03%	-0.30%
Z2	+3.08%	-0.32%	+2.78%	+6.98%	-1.45%	-0.62%	-5.89%
Z3	+0.15%	-0.57%	+0.66%	+2.17%	-1.52%	-0.56%	-1.45%
Z4	-0.07%	+0.50%	-0.57%	-2.00%	+1.38%	+0.55%	1.27%
Z5	+0.05%	-0.31%	+0.71%	+1.02%	-0.80%	-0.28%	-0.57%
Z6	+0.31%	-0.17%	+1.11%	+2.08%	-0.71%	-0.49%	-1.07%
Z7	+0.32%	-0.24%	+1.07%	+2.18%	-0.85%	-0.52%	-1.16%
Z8	+1.24%	-0.12%	+1.86%	+3.65%	-0.88%	-0.59%	-2.60%
Z9	-0.84%	+0.94%	-2.78%	-9.06%	+4.27%	+2.49%	5.45%
Z10	+0.68%	-0.52%	+1.20%	+4.14%	-1.59%	-1.12%	-2.70%

2.2 Lifting 1

<i>Risk figure</i>	<i>Modification of the yield curve</i>
τ_1	Lifting by $\log(1,0+t)/100$ percent, starting at the overnight rate.
τ_2	Lifting by $\log(1,0+t-2,0)/100$ percent, starting at two years.
τ_3	Lifting by $\log(1,0+t-5,0)/100$ percent, starting at five years.

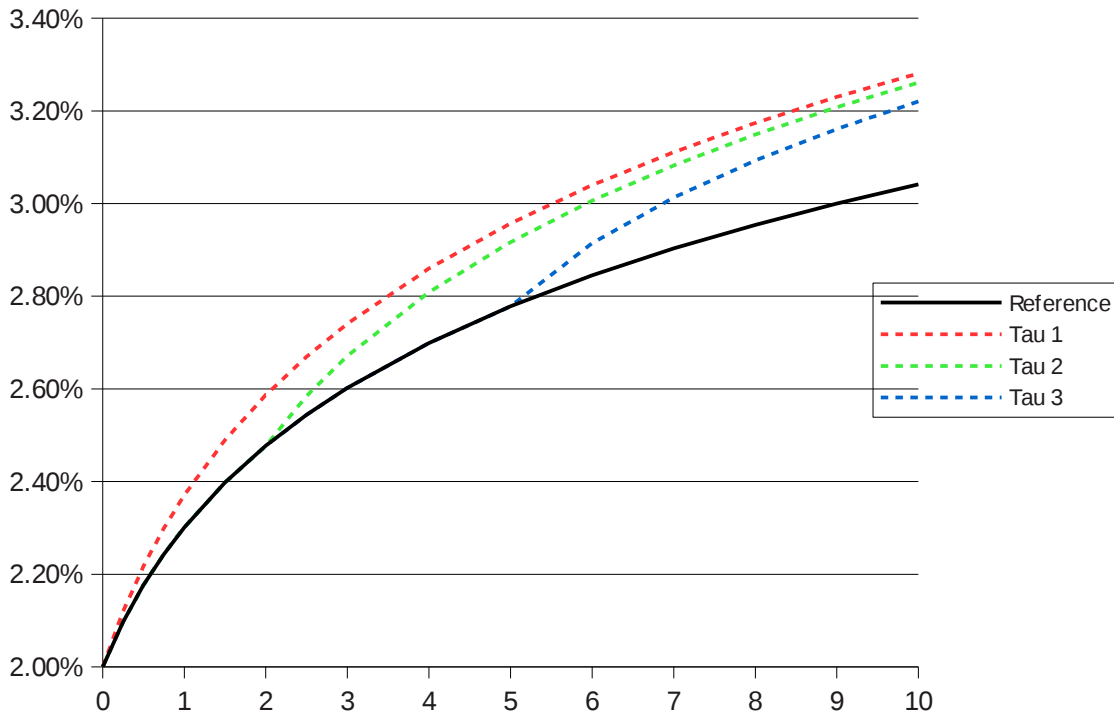


Figure 3: Lifting 1

	A	B	C	D	E	F	G
Z1	+0.45%	-0.06%	+2.33%	+0.21%	-0.20%	-0.14%	-0.27%
Z2	+4.43%	-0.37%	+13.43%	+4.78%	-1.22%	-0.79%	-4.87%
Z3	+0.44%	-0.35%	+2.44%	+0.17%	-0.59%	-0.24%	-0.48%
Z4	-0.05%	+0.23%	-0.13%	-0.03%	+0.34%	+0.11%	0.30%
Z5	+0.08%	-0.17%	+0.72%	+0.01%	-0.24%	-0.07%	-0.07%
Z6	+0.31%	-0.06%	+2.28%	+0.06%	-0.18%	-0.13%	-0.07%
Z7	+0.06%	-0.04%	+0.01%	+0.05%	-0.06%	-0.01%	-0.10%
Z8	+1.78%	-0.10%	+7.38%	+1.26%	-0.55%	-0.41%	-1.45%
Z9	-0.28%	-0.01%	-0.60%	+0.05%	+0.50%	+0.43%	0.92%
Z10	+0.56%	-0.28%	+2.55%	+0.16%	-0.49%	-0.36%	-0.74%

2.3 Lifting 2

<i>Risk figure</i>	<i>Modification of the yield curve</i>	
τ_{1}	Lifting by $\log(1,0+5,0-t)/100$	percent up to five years.
τ_{2}	Lifting by $\log(1,0+t-2,0)/100$	percent, starting at two years.
τ_{3}	Lifting by $\log(1,0+t-5,0)/100$	percent, starting at five years.

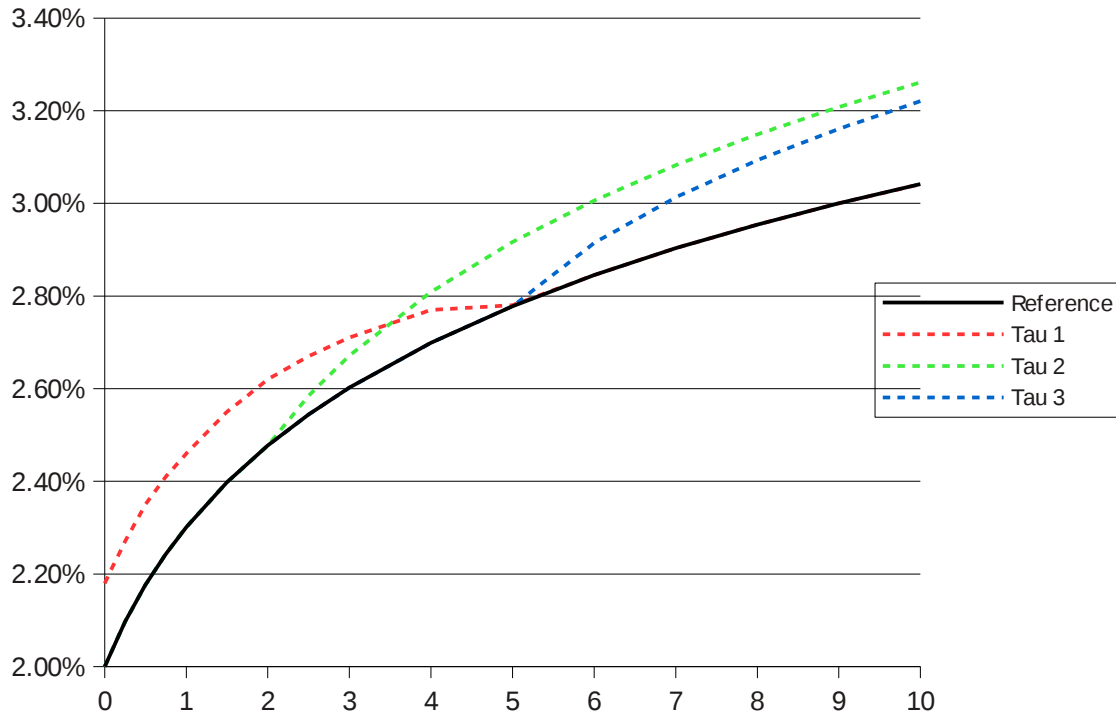


Figure 4: Lifting 2

	A	B	C	D	E	F	G
Z1	-1.18%	-0.43%	-0.77%	-0.06%	-0.39%	+0.81%	+2.19%
Z2	-3.42%	-2.12%	-1.49%	+3.47%	-2.14%	+3.80%	+6.98%
Z3	-0.88%	-0.65%	-0.06%	-0.05%	-0.75%	+0.52%	+1.51%
Z4	-0.38%	+0.16%	-0.74%	-0.09%	+0.30%	+0.30%	+0.78%
Z5	+0.08%	-0.17%	+0.71%	+0.01%	-0.24%	-0.06%	-0.06%
Z6	-0.55%	-0.26%	+0.64%	-0.09%	-0.28%	+0.37%	+1.23%
Z7	+0.84%	+0.13%	+1.50%	+0.18%	+0.04%	-0.47%	-1.28%
Z8	-2.29%	-1.01%	-0.35%	+0.58%	-1.03%	+1.96%	+4.68%
Z9	-1.89%	-0.37%	-3.66%	-0.22%	+0.31%	+1.37%	+3.35%
Z10	-0.43%	-0.50%	+0.66%	-0.01%	-0.60%	+0.22%	+0.76%

2.4 Rho-Lifting

<i>Risk figure</i>	<i>Modification of the yield curve</i>
τ_1	Parallel shift of 0.01% upwards
τ_2	Lifting by $\log(1,0+t-2,0)/100$ percent, starting at two years.
τ_3	Lifting by $\log(1,0+t-5,0)/100$ percent, starting at five years.

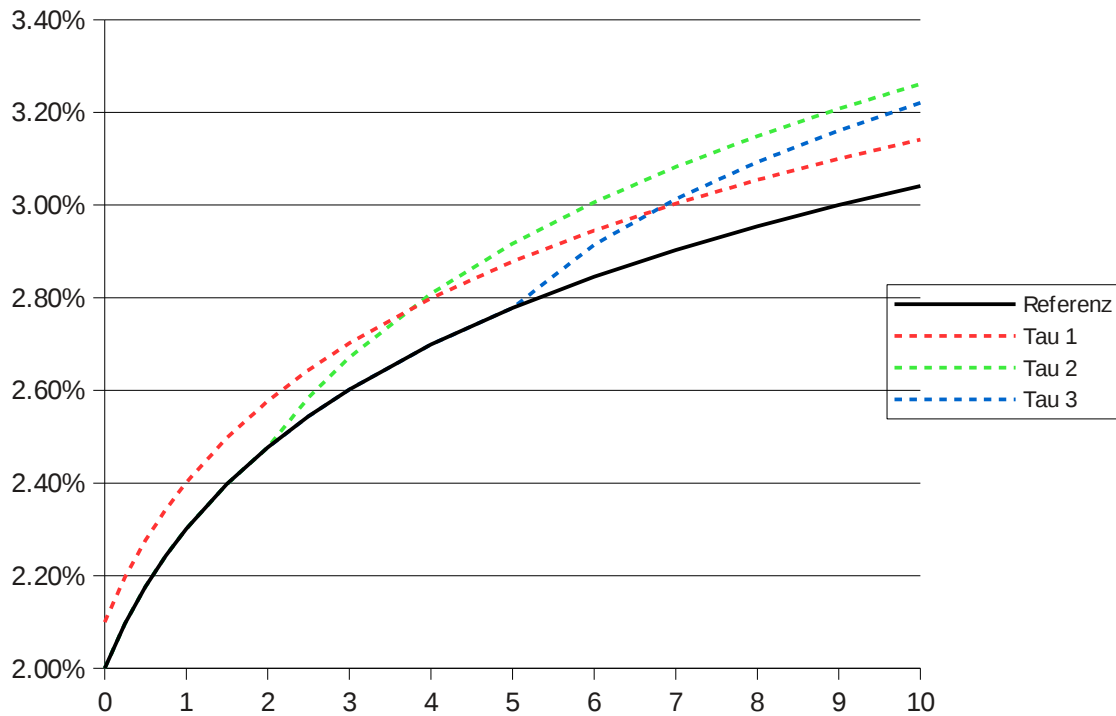


Figure 5: Rho-and-Lifting

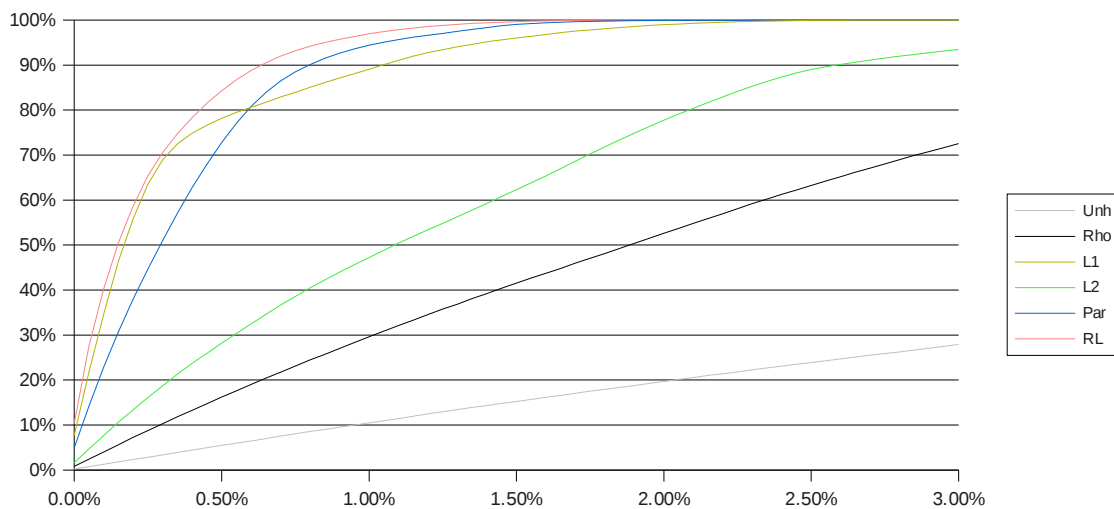
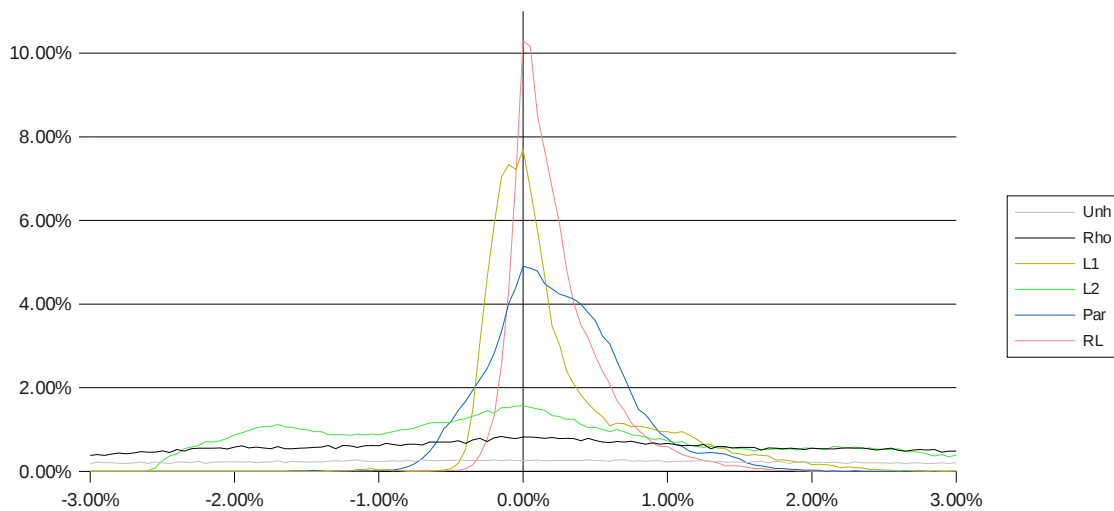
	A	B	C	D	E	F	G
Z1	+0.17%	+0.00%	+0.12%	+0.20%	-0.02%	-0.01%	-0.26%
Z2	+3.11%	-0.05%	+2.78%	+4.75%	-0.38%	-0.14%	-4.79%
Z3	+0.22%	-0.30%	+0.66%	+0.17%	-0.45%	-0.14%	-0.46%
Z4	-0.11%	+0.25%	-0.57%	-0.03%	+0.38%	+0.13%	0.30%
Z5	+0.08%	-0.17%	+0.71%	+0.01%	-0.24%	-0.07%	-0.07%
Z6	+0.17%	-0.03%	+1.11%	+0.05%	-0.09%	-0.06%	-0.07%
Z7	+0.19%	-0.07%	+1.07%	+0.06%	-0.14%	-0.07%	-0.11%
Z8	+1.09%	+0.06%	+1.86%	+1.24%	-0.11%	-0.08%	-1.41%
Z9	-0.55%	+0.06%	-2.78%	+0.05%	+0.67%	+0.56%	0.94%
Z10	+0.40%	-0.24%	+1.20%	+0.15%	-0.38%	-0.28%	-0.73%

2.5 Verification based on historical yield curves

In this section the different approaches are verified based on historical yield curves. In order to do so for each week in the period from January 4th 1999 to January 19th 2006 the historical yield curve will be determined. Based on each of those yield curve the derivatives are valued and different hedges are constructed according to the presented approaches. Each hedged portfolio is then revalued with each other possible yield curve and the relative change in value of the hedged portfolio is determined. Finally the distribution of the portfolio value changes are calculated for each approach.

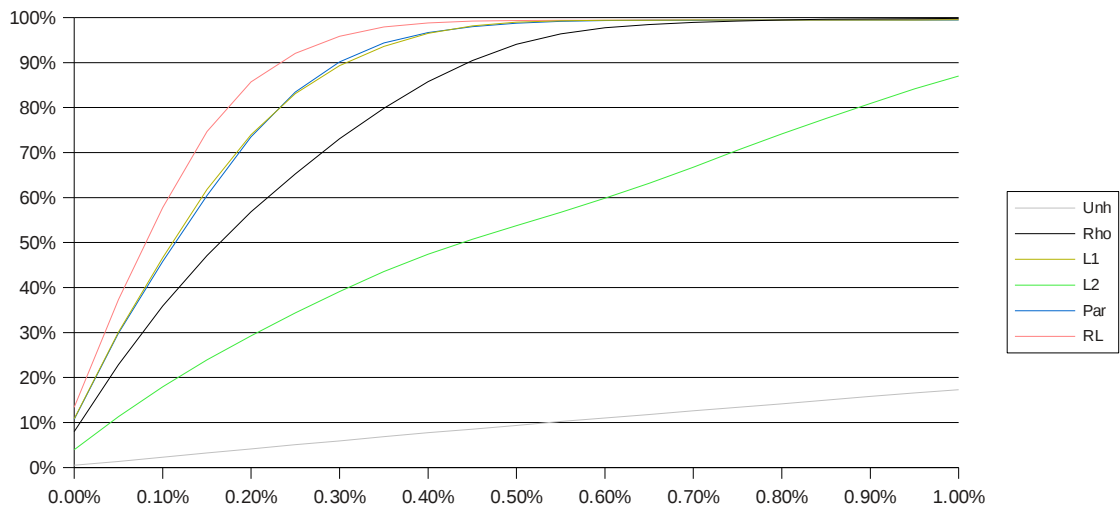
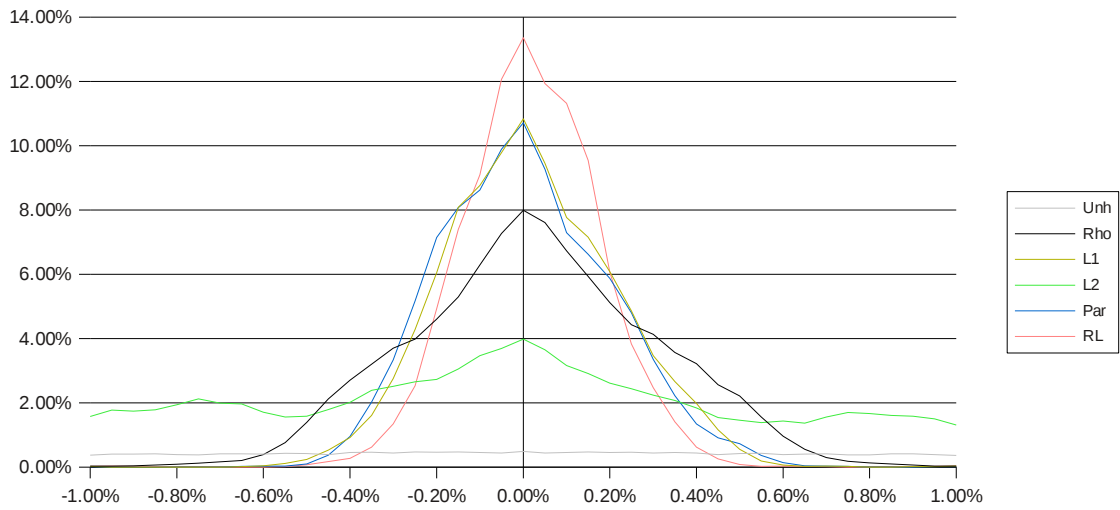
2.5.1 Asian call option with a maturity of six years

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.4491%	0.4837%	0.2418%	0.2422%	0.2151%	0.2302%
Std. dev.	9.5166%	2.7153%	0.4562%	0.5396%	1.6210%	0.3225%



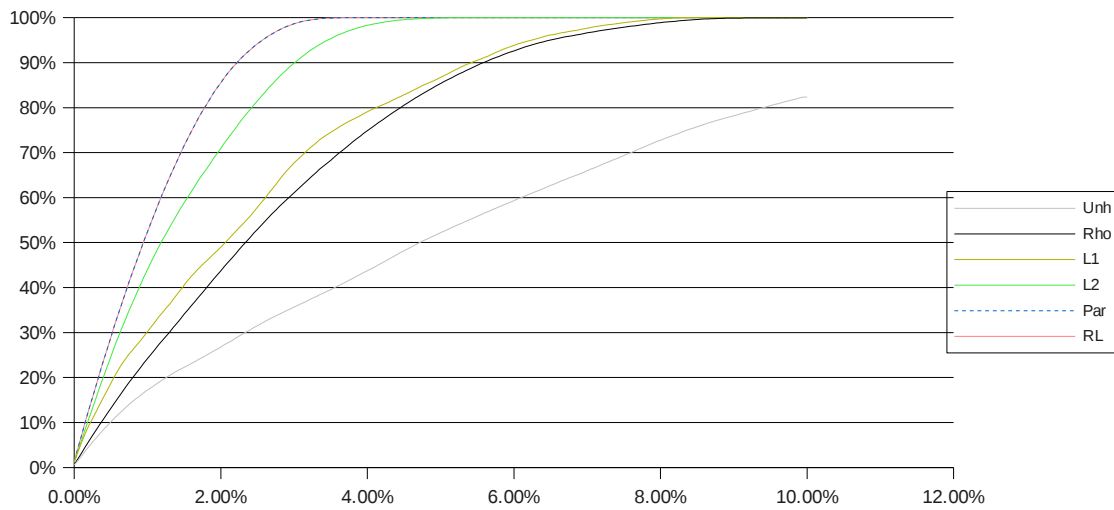
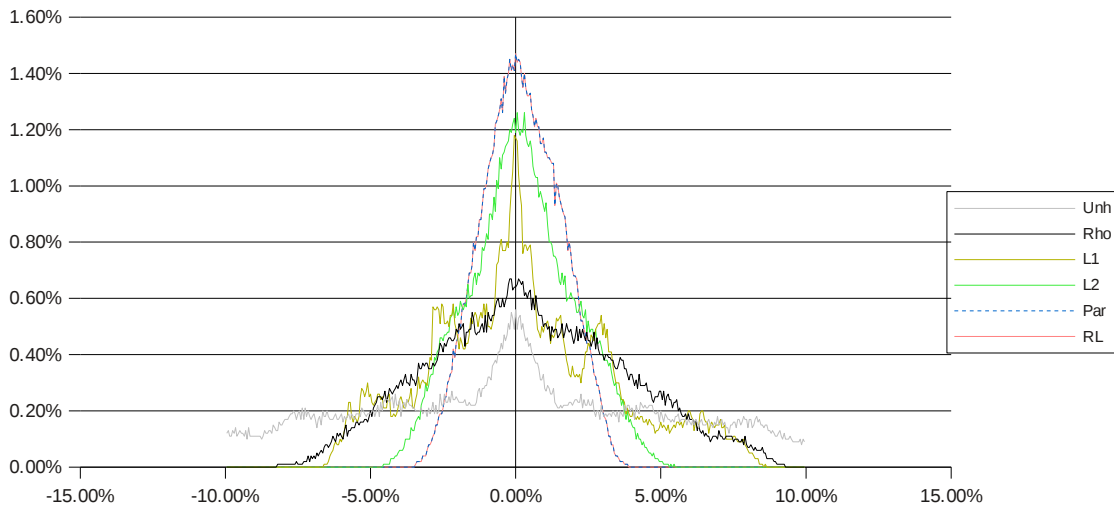
2.5.2 Winner

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.1298%	0.0277%	0.0039%	0.0148%	0.0276%	0.0196%
Std. dev.	5.1017%	0.2859%	0.2165%	0.2147%	0.6666%	0.1741%



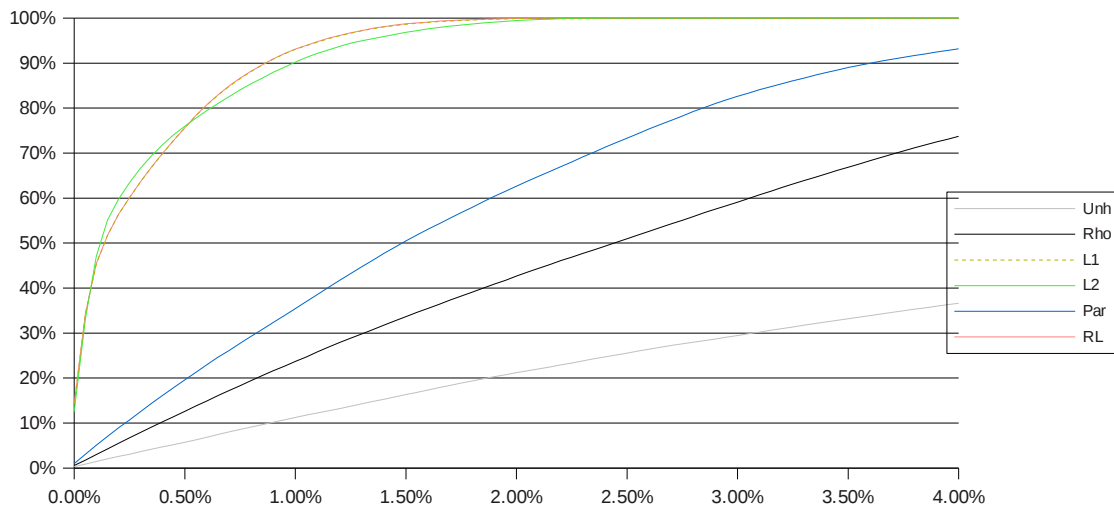
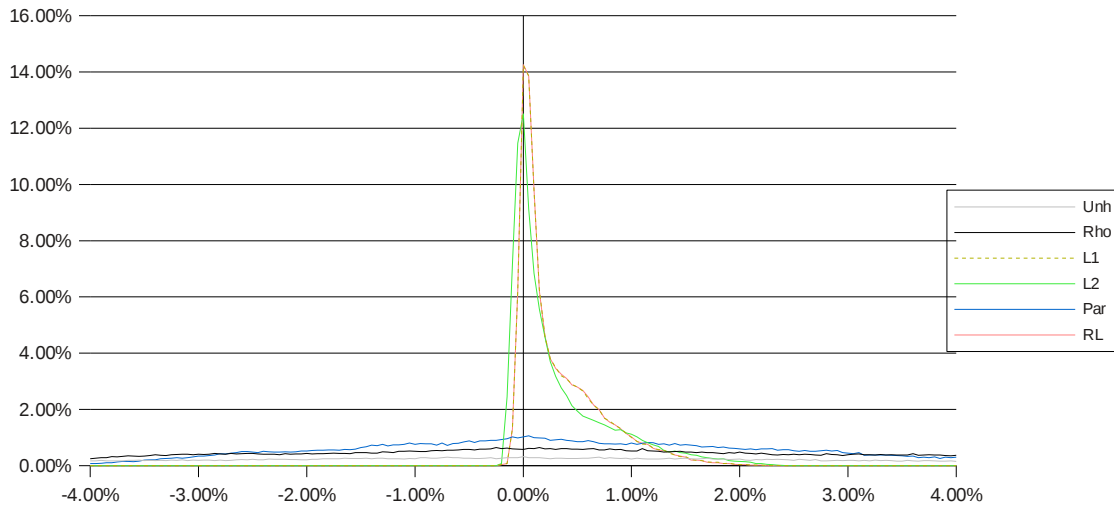
2.5.3 Asian call option with a maturity of 18 months

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.2404%	0.4240%	0.1978%	0.2106%	0.1922%	0.1978%
Std. dev.	6.9501%	3.3477%	1.3402%	3.1247%	1.7974%	1.3402%



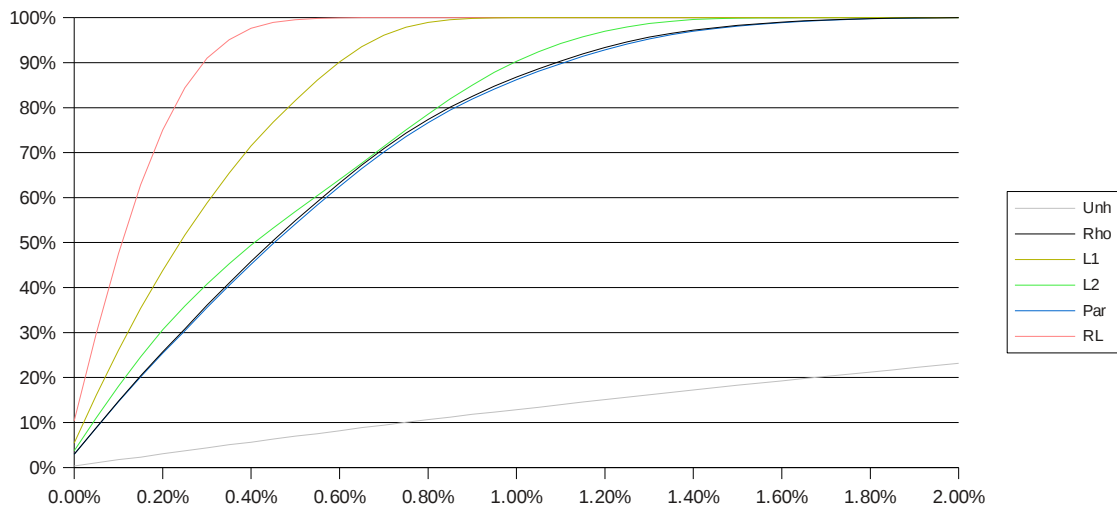
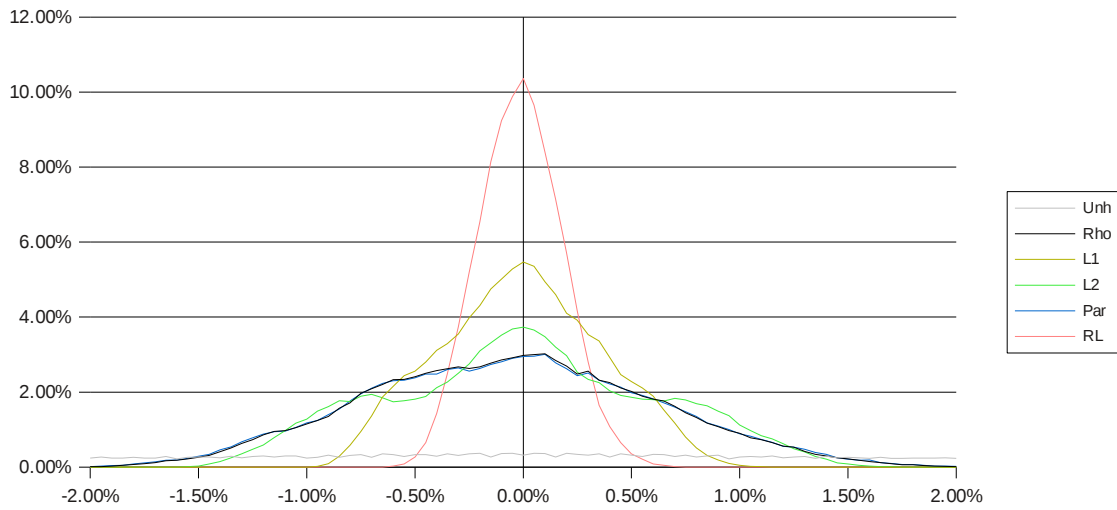
2.5.4 Call

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.4265%	0.6239%	0.4625%	0.3210%	0.3157%	0.3203%
Std. dev.	9.2731%	3.5083%	2.1818%	0.3911%	0.4663%	0.3891%



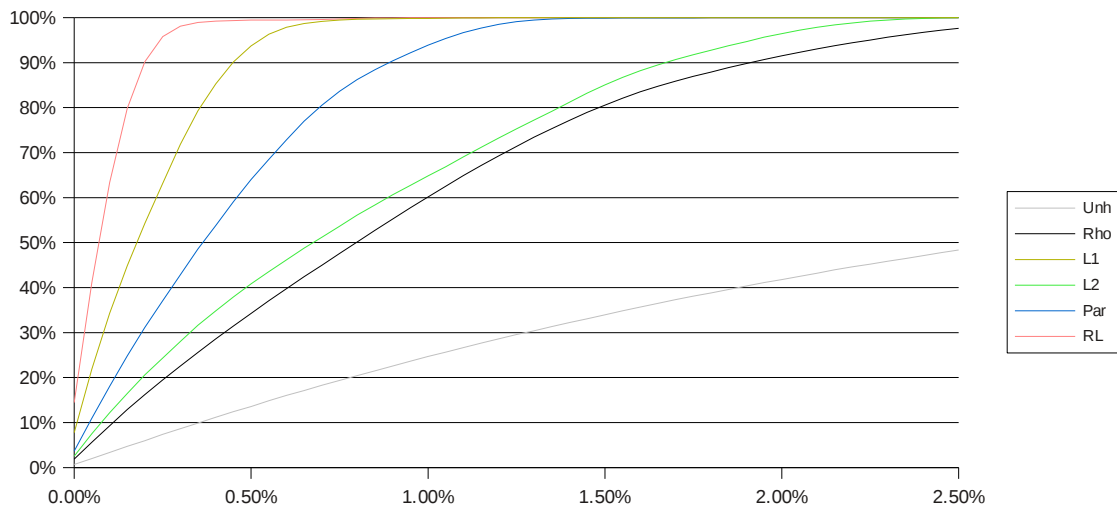
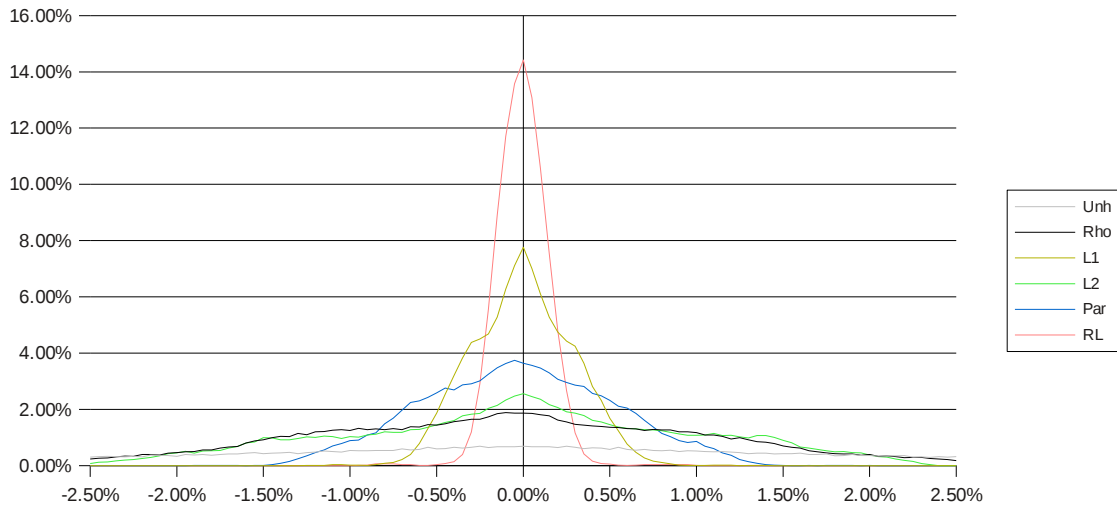
2.5.5 Bond

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.2570%	-0.0621%	-0.0623%	-0.0103%	-0.0073%	-0.0120%
Std. dev.	7.1861%	0.6667%	0.6776%	0.3724%	0.6146%	0.1919%



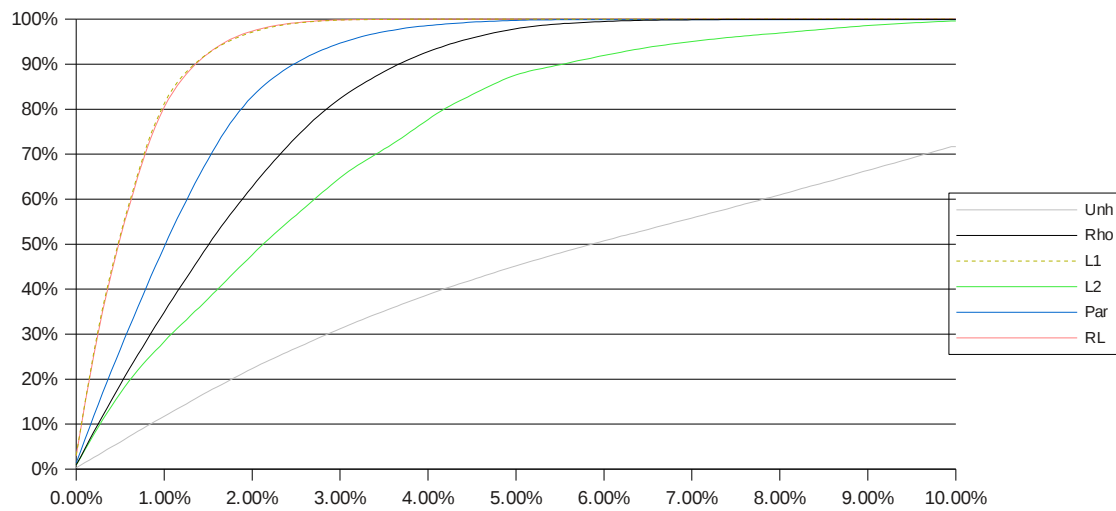
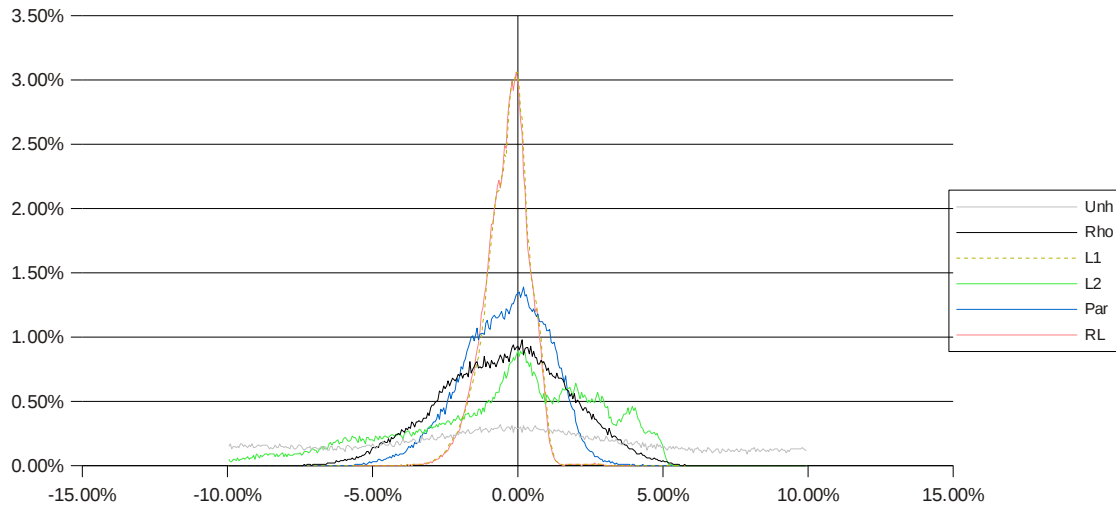
2.5.6 Zero coupon bond

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.0735%	-0.0889%	-0.0310%	-0.0033%	-0.0190%	-0.0053%
Std. dev.	3.8369%	1.1546%	0.5465%	0.2905%	1.0133%	0.1468%



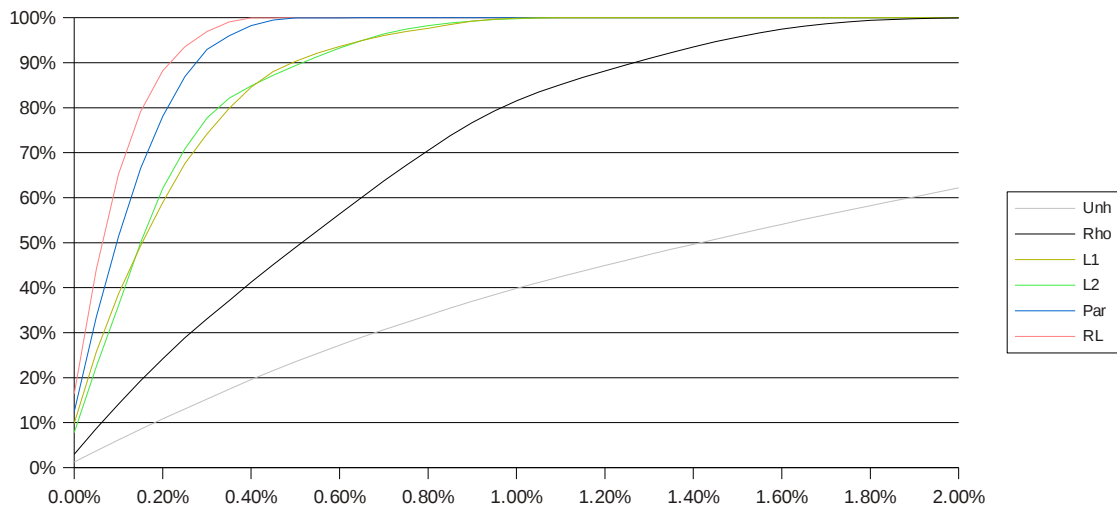
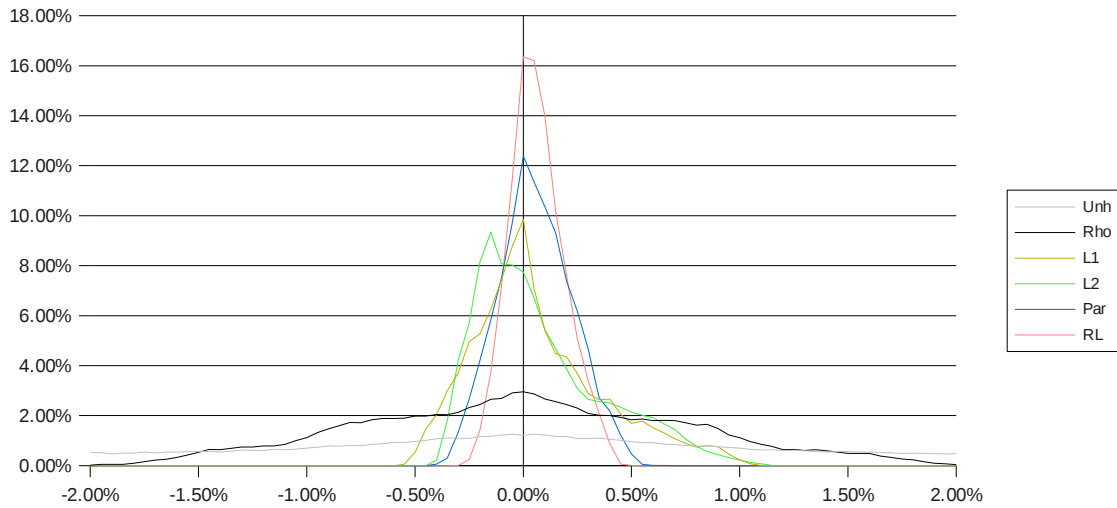
2.5.7 Down-and-out put

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.3549%	-0.4328%	-0.4161%	-0.3516%	-0.5477%	-0.3764%
Std. dev.	8.4528%	2.1864%	1.4859%	0.7611%	3.3447%	0.75020%



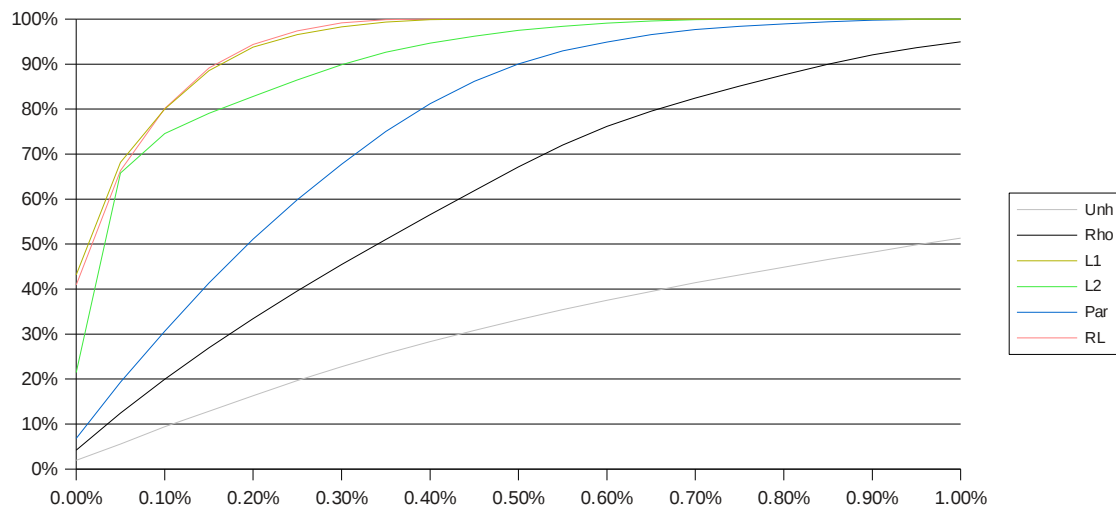
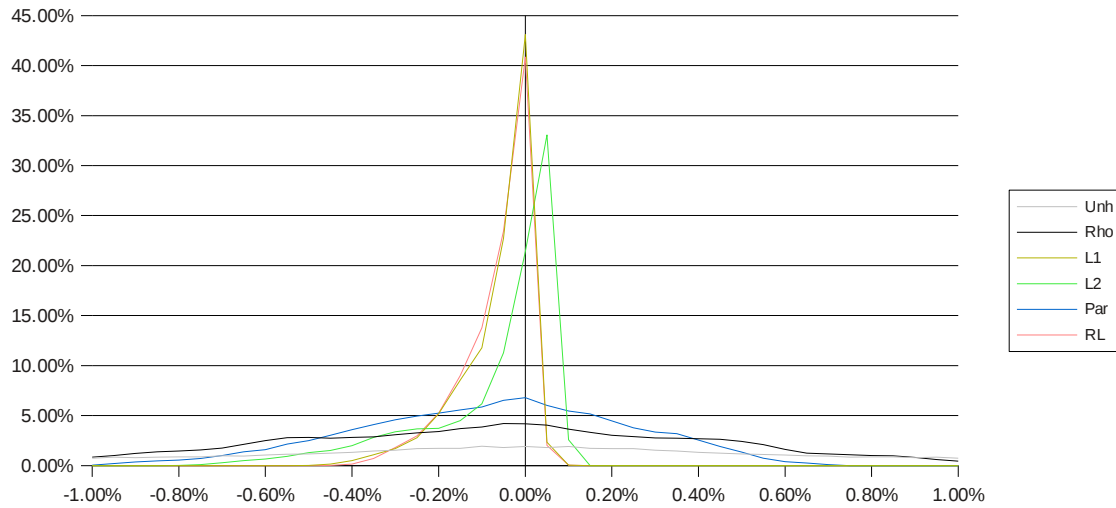
2.5.8 Auto-callable

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0,0216%	0,0029%	0,0591%	0,0655%	0,0772%	0,0649%
Std. dev.	2,0775%	0,7606%	0,1697%	0,3048%	0,2975%	0,1255%



2.5.9 Discount certificate

	<i>Unhedged</i>	<i>Rho</i>	<i>Parallel</i>	<i>Lifting 1</i>	<i>Lifting 2</i>	<i>Rho-Lifting</i>
Mean	0.0102%	-0.0912%	-0.0757%	-0.0636%	-0.0723%	-0.0648%
Std. dev.	1.4254%	0.5176%	0.3121%	0.0857%	0.1642%	0.0796%



2.6 Conclusion

Based on the previous verifications the approach Rho-Lifting is by far the superior approach. It provided in all test cases with historical yield curves the best results and performed above average in the hypothetical scenarios.

In the examples in section 1 the change in portfolio value for the asian option would have been 1,500€ if hedged with the approach Rho-Lifting, compared to 40,100€ if hedged with a simple Rho hedge. For the Down-and-out put the change in portfolio value would have been 2,000€ compared to 25,000€.